water.surf.transfer-rs.hayami

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Name : *Water transfer on reach segments using hayami propagation method*
Version : *13.05*
Domain : *hydrology*
Description : *Calculation of discharge routing through the ditch network using diffusive wave equation resolved with Hayami method and discharge convertion to water height*

Simulator parameter(s)

`calibstep`	calibration step for height-discharge relation	m
`maxsteps`	maximum hayami kernel steps	$-$
`meancel`	wave mean celerity on RSs	m/s
`meansigma`	mean diffusivity on RSs	$m2/s$
`rsbuffer`	buffer upon reach for water height over reach height	m

Required or used Attribute(s)

`height`	required	RS	height of the RS	m
`length`	required	RS	lenght of the RS	m
`nmanning`	required	RS	Manning roughness coefficient of the RS	$s/m(-1/3)$
`slope`	required	RS	mean slope of the RS	m/m
`width`	required	RS	width of the RS	m

Required or used variable(s)

`water.surf.Q.downstream-su`	used	SU	output volume at the outlet of the SU	m3/s
`water.sz-surf.Q.baseflow`	used	GU	drainage output volume from GU to connected RS	m3/s
`water.uz.Q.interflow`	used	SU	interflow output volume at the outlet of the SU soil reservoir	m3/s

Produced or updated variable(s)

`water.surf.H.level-rs`	produced	RS	water height at the outlet of the RS	m
`water.surf.Q.downstream-rs`	produced	RS	output volume at the outlet of the RS	$m3/s$

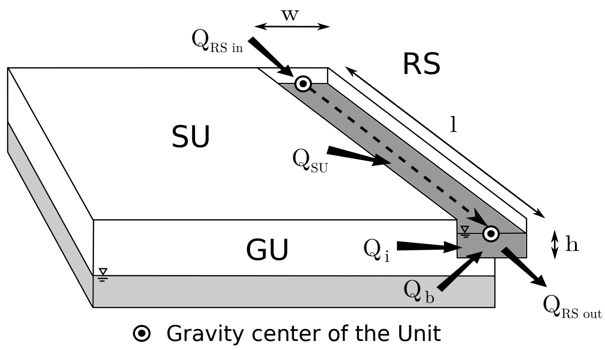
Abstract

The "Water transfer on ditch network using hayami propagation method" compute the water transfer through the ditch network using the diffusive wave model resolved by the Hayami method. First, it calculates the input volume in reach segments. Then, the input hydrogram is convoluted using diffusive wave model with celerity and diffusivity as main parameters. Finally, the simulator converts the calculated output discharge to water height in the ditch using Manning-Strickler equation.

1 Scientific concepts

1.1 Calculation of input water discharge

The simulator transfers water in hydrographic channel from upstream RS to discharge system RS. At each node, upper hydrograms are cumulated and propagated to downstream RS.



The first step consists in computing the input discharge at the inlet of the ditch. This value is composed with discharge from potential connected RS or SU and subsurface and aquifer drainage flow if the model previously produces these variables. The equation is the following :

$$Q_{in} = \sum_{RS_{Up}} Q_{RS} + \sum_{SU_{Up}} (Q_{SU} + Q_i) + \sum_{GU_{Up}} Q_b \quad (1)$$

where Q_in is the input water discharge at the inlet of the RS (m^3/s), Q_{RS} is the discharge produced by the upper connected RS, Q_{SU} is the discharge from the upper SU, Q_b is the baseflow discharge from connected GU (m^3/s), and Q_i is the interflow discharge (m^3/s).

1.2 Propagation using Hayami kernel

Then, the input water discharge previously calculated is propagated between the gravity center of the main SU and the downstream unit. The propagation is done using the diffusive wave model [?] :

$$\frac{\delta Q}{\delta t} = -C \times \frac{\delta Q}{\delta x} + D\delta \frac{\delta^2 Q}{\delta x^2} \quad (2)$$

In the particular case where wave celerity and diffusivity are constant, the diffusive wave equation can be resolved using an analytical solution [?]. The water output discharge is calculated using the following equation :

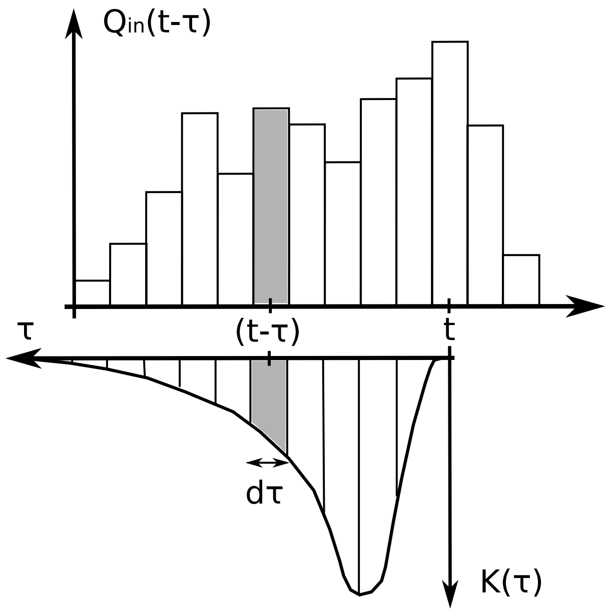
$$Q_{RS}(t) = Q_{in}(t) * K(t) \quad (3)$$

where Q_{RS} is the produced output water discharge (m^3/s), Q_{in} is the input water discharge on RS (m^3/s) calculated in the equation 1, $*$ is the convolution product, and $K(t)$ is the “Hayami kernel” function expressed as :

$$K(t) = \frac{L}{2 \times (\pi D)^{\frac{1}{2}}} \times \frac{exp^{\frac{CL}{4D} \times \left(2 - \frac{L}{Ct} - \frac{Ct}{L}\right)}}{t^{\frac{3}{2}}} \quad (4)$$

where L is the length of RS (m), D is the wave diffusivity (m^2/s), and C is the wave celerity (m/s).

To resolved the equation 3, the simulator temporarily sweeps the Hayami kernel. τ is a convolution time with $\delta\tau$ wich is equal to the simulation time step. This operation can be schematized as following :



The $K(t)$ function of Hayami Kernel is defined on $[0 + \infty]$. However, the kernel covolution cannot be done up to the infinity. $MaxSteps$ parameter $(-)$ defines the number of time step and, consequently, the limit duration $MaxSteps.\Delta t$. It should be adapted according to the kernel shape and the simulation time step in the way to minimise the kernel truncation. The higher this parameter is, the better the definition of Hayami kernel is. Moreover, the smaller the simulation time step is, the better the Hayami kernel resolution is. The optimal way to parametrise the model for a given simulation time step is to start from a height value of $MaxSteps$ and then dicreases it while no change on results.

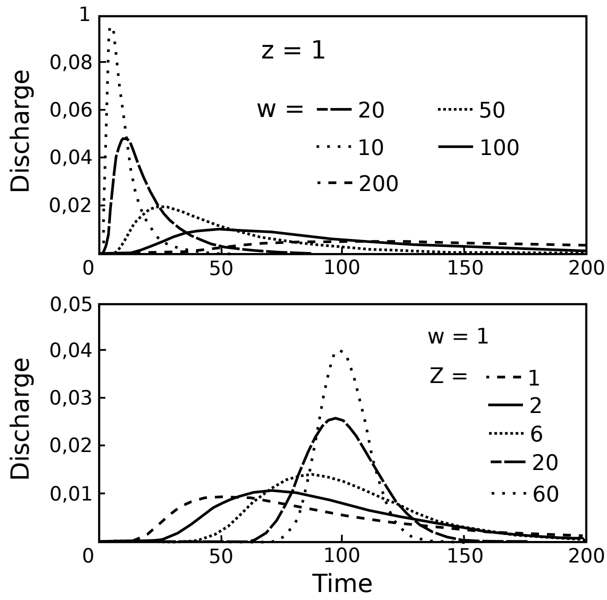
The two main parameters C and D of the diffusive model could be related to the slope and the rugosity of the surface unit using Manning-Strickler relation.

$$C = C_u \times \sqrt{\frac{\beta}{\beta_m}} \times \frac{n_m}{n} \quad et \quad D = D_u \times \frac{\beta}{\beta_m} \times \frac{n_m}{n} \quad (5)$$

where C_u is the mean wave celerity (m/s), β is the slope of the RS on which the calculation is done (m/m), β_m is the mean slope of RSs (m/m), n is the rugosity coefficient of the RS ($s/m^{1/3}$), n_m is the mean rugosity coefficient of RSs ($s/m^{1/3}$) and D_u is the mean wave diffusivity (m^2/s). This calculation is done once at the beginning of the simulation.

Diffusivity and celerity define the shape of Hayami unit hydrogram as :

$$w = \frac{L}{D} \qquad z = \frac{C.L}{4.D} \quad (6)$$



Examples of use and parametrization of this model are available in the thesis of Chahinian [**?**] and a paper of Moussa and al. [**?**].

1.3 Discharge - Height conversion

Then, the simulator converts the previous calculated discharge to water height in the reach segment. This convertion is done using the Manning-Strickler equation which links height and discharge for a rectangular ditch section :

$$Q_{RS} = \frac{1}{n} \times \sqrt{\beta} \times R^{\frac{2}{3}} \times l \times h \quad with \quad R = \frac{l \times h}{l + 2h} \quad (7)$$

where n is the rugosity coefficient of RS ($s/m^{1/3}$), β is the slope of RS (m/m), R is the hydraulic radius of the ditch (m) considered as a rectangular section, l is the width of RS (m), and h is the water height in the reach segment (m) corresponding to discharge.

The simulator compute height-discharge relation using the previous equation, once for each RS at the begining of the simulation. Then, the obtained curve is used to determine for each time step the water height corresponding to the calculated output discharge Q_{RS}. The calibration curve of the RS is computed with a "height step" given by the *CalibStep* parameter (in m). The smaller this parameter is, the more accurate the calculated water height is. This relation is calculated up to a maximum height which is the RS height H plus the $RSbuffer$ parameter (in m). Up to this value, the simulator displays a warning which indicated that the RS water height is out of range.

2 Functional description

2.1 Simulator name

The name (ID) of the simulator is `water.surf.transfer-rs.hayami`.

2.2 Simulator parameters

The simulator "Water transfer on ditch network using hayami propagation method" must be used with the following parameters :

Symbol	*Name*	*Value range*	*Unit*
$MaxStep$	`maxsteps`	> 0	$-$
C	`meancel`	> 0	m/s
D	`meansigma`	> 0	m^2/s
$CalibStep$	`calibstep`	> 0	m
$RSbuffer$	`rsbuffer`	≥ 0	m

Thus, the correct syntax to use in the `model.xml` file is illustrated hereafter.

```
<simulator ID="water.surf.
          transfer-rs.hayami">
    <param name="calibstep" value="0.005" />
    <param name="maxsteps" value="100" />
    <param name="meancel" value="0.5" />
    <param name="meansigma" value="500" />
    <param name="rsbuffer" value="1" />
</simulator>
```

2.3 Unit properties required

The simulator requires some geometric properties and soil characteristics. These are described in the following table.

Symbol	*Name*	*Value range*	*Unit*
n	`nmanning`	> 0	$s/m^{-1/3}$
L	`length`	> 0	m
l	`width`	> 0	m
H	`height`	> 0	m
β	`slope`	> 0	m/m

2.4 Variables

Variables produced, required and updated by the simulator are listed hereafter.

Symbol	*Name*	*Unit*
Q_{SU}	`water.surf.Q.downstream-su`	m^3/s
Q_b	`water.sz-surf.Q.baseflow`	m^3/s
Q_i	`water.uz.Q.interflow`	m^3/s
Q_{RS}	`water.surf.Q.downstream-rs`	m^3/s
h	`water.surf.H.level-rs`	m

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